Connection between random networks and nonextensive statistical mechanics

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Boltzmann-Gibbs statistical mechanics, based on the additive BG entropy, exhibits various isomorphisms with random systems, e.g., the Kasteleyn-Fortuin theorem for bond percolation, the de Gennes isomorphism for self-avoiding random walk, equivalence with random networks of impedances. Similarly, nonextensive statistical mechanics, based on the nonadditive entropy S_q, is connected (see [1-3] and references therein) with the so-called (asymptotically) scale-free random networks. We present here a computational study [3] of one such d-dimensional geographic model with a preferential-attachment probability decaying like $1/distance^{\alpha_A}$ ($\alpha_A \geq 0$) with links weighted through the distribution $P(w) \propto e^{-(w/w_0)^\eta}$ ($w_0 > 0$, $\eta > 0$). We exhibit that the distributions of energy ϵ are very satisfactorily fitted by q-exponentials $e_q^{-\beta_q \epsilon} \equiv 1/[1+(q-1)\beta_q \epsilon]^{1/(q-1)}$ ($q \geq 1$, $\beta_q > 0$), which optimize the entropy S_q under simple constraints. In Fig. 1 we illustrate the model, the quality of the fittings, and the α/d-dependence of (q, β_q). Notice that the dependences on (α_A, d) only act through the ratio α_A/d, and that the entropic index q does not depend on (η, w_0), thus characterizing universality classes.

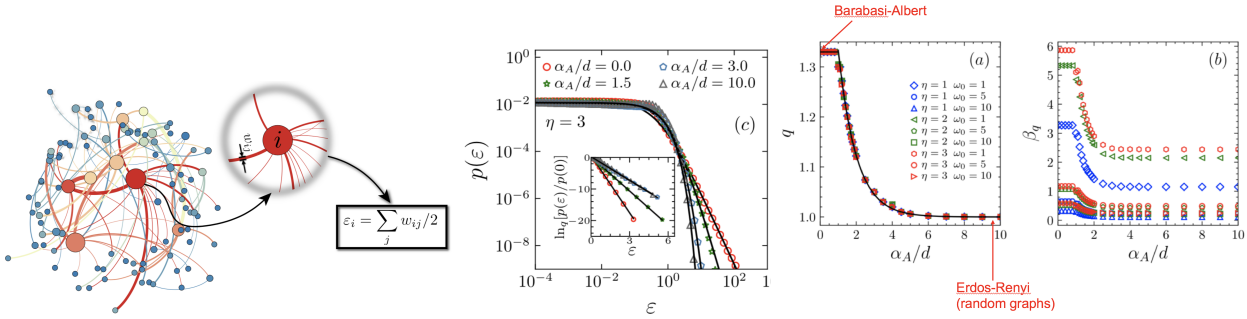

Figura 1: *Left: Model. Center: Energy distributions fitted by q-exponentials. Right:* (q, β_q) *parameters as functions of* (α/d).

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References

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